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Note: Entrant's Product Quality Signaling through a Trial Period in a Competitive Market

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This note provides a self-contained analysis of an entrant-incumbent competition as an extension of the monopolistic trial period signaling model examined by Wang and Özkan-Seely (2016). Our goal is to capture the tension between the entrant's need to penetrate the incumbent's market and its signaling desire to convey private information about its product quality. The interplay between competition and signaling significantly complicates the analysis of such games of incomplete information; and probably for this reason, examples of such games with analytically tractable solutions are quite sparse in the literature. Despite those challenges, in this note we develop a tractable model by starting with the trial period's learning and dispersion effects identified in Wang and Özkan-Seely (2016) and characterize the entrant's equilibrium trial strategies. As a key result, we find that all the separating equilibria entail the high-quality entrant offering a *longer* trial period than its low-quality counterpart, which is consistent with our finding in the monopolistic case (Wang and Özkan-Seely 2016). We also identify a sufficient parametric condition, under which the low-quality entrant offers no trial period in the equilibrium, recovering the exact result as in Wang and Özkan-Seely's (2016) monopolistic model.

We consider a setting where a new entrant firm launches a product of unknown quality in a market occupied by an incumbent firm. Selling a product of publicly observable quality, the incumbent does not offer any trial period, whereas the entrant can commit to a trial period of a certain length. Both firms can adjust their prices after observing the entrant's trial length. Therefore, only the trial length serves as a quality signal for the entrant, and both firms engage in a price competition. This type of incumbent-entrant model helps disentangle the signaling effects from the competition effect and has precedence in the literature (e.g., Balachander 2001).

We first examine the complete-information benchmark, whereby the entrant's product quality is publicly observable. Without the need for a quality signal, this benchmark case identifies the impact of competition. We find that the entrant with a significant quality advantage over the incumbent offers no trial period whereas it is optimal for the entrant without such quality advantage to offer the longest possible trial period (Proposition 2). This finding partly parallels that of the the monopolistic case, where a firm of *any* publicly observable quality level prefers no trial period. In essence, a monopolistic firm competes with a no-purchase default option, whereas an entrant in a competitive market competes with an incumbent of known quality, who *strategically* prices its product. A longer trial period leads to higher consumer heterogeneity and spread out the posterior distribution of consumer's WTP due to the dispersion effect, which in turn softens the competition with the incumbent. While the entrant without a sufficient quality advantage over the incumbent always has such an incentive to obfuscate its quality disadvantage, the entrant with a high enough quality level nonetheless avoids doing so to minimize the dispersion effect.

In our focal case, the entrant's product quality is its private information, and the entrant can potentially distinguish its quality type by committing to its trial strategy (i.e., the trial length and price). Observing the entrant's trial length, both the incumbent and the market form their beliefs about the entrant's product quality and react accordingly. In particular, the entrant and incumbent engage in price competition. Taking into account such strategic responses, we derive the entrant's profit function in the reduced form, allowing us to characterize the incentive compatibility constraints defining the separating equilibria.

In a key result (Proposition 3), all the separating equilibria are shown to feature a *longer* trial length for the high-quality entrant than for the low-quality entrant, consistent with the qualitative finding obtained in the monopolistic case (Wang and Özkan-Seely 2016). In particular, a low-quality entrant with sufficient quality advantage over the incumbent offers *no* trial period (i.e., zero trial length), recovering the exact result obtained in Wang and Özkan-Seely (2016). Meanwhile, the low-quality entrant without such obvious quality advantage may offer a trial period of positive length (but still shorter than that of the high-quality entrant), which is distinct from the monopolistic result. As alluded to above, this is because the low-quality entrant has an incentive to leverage the trial period's dispersion effect to soften the competition with the incumbent. The need for a quality signal, in turn, forces the high-quality entrant to offer an even longer trial period than that of the low-quality entrant *even if it has a significant quality advantage over the incumbent*, which is quite different from the above-mentioned complete-information benchmark case. On the other hand, when the low-quality entrant has sufficient quality advantage over the incumbent, it does not need to soften the competition with the incumbent and hence offers no trial period to minimize the dispersion effect.

1. Model Description

We consider a duopoly model in reduced form, where a new entrant firm launches a new product of unknown quality q_e in a market dominated by an incumbent firm of known product quality q_i. Consumers have been relatively familiar with the incumbent's product without any product trial and has a heterogeneous willingness-to-pay (WTP) of

$$X_i(q_i) := q_i + \epsilon_i, \tag{1.1}$$

where the random variable ϵ_i is assumed to have mean zero. The incumbent's price is denoted as p_i.

Consumers believe that the entrant's product quality q_e can be either high $\bar{q}$ with probability λ or low $\underline{q}$ with probability $1-\lambda$, where we assume $\bar{q} > q_i$ whereas $\underline{q}$ can be higher or lower than q_e (e.g., Balachander 2001). The entrant's price is denoted as p_e. The entrant commits to offer a trial period of length $t_e \in [0,1]$.

Nonetheless, neither the entrant nor the incumbent is able to commit to the price *a priori* (see, for example, Dmitri and Lin (2010) for a similar assumption and justification). The sequence of events evolves as follows. The entrant firm first decides and announces its trial length t_e, from which the incumbent and the market infer the entrant's product quality to be $\widehat{q}_e = \widehat{q}_e(t_e) \in \{\underline{q}, \bar{q}\}$. We will only consider separating equilibria. Subsequently, both the entrant and the incumbent adjust their prices and conduct a price competition. Therefore, only the trial length can serve as the incumbent's quality signal.

At the conclusion of the trial period, the consumer's WTP for the entrant's product becomes

$$X_e(t_e, q_e, \widehat{q}_e) := \mu(t_e, q_e, \widehat{q}_e) + \xi_e(t_e), \tag{1.2}$$

where $\mu(t_e, q_e, \widehat{q}_e) := t_e q_e + (1-t_e)\widehat{q}_e$ gradually shifts from $\widehat{q}_e$ to q_e as t_e increases from 0 to 1, capturing the trial period's *learning effect*, and the random variable $\xi_e(t_e)$ is assumed to have mean zero and larger variance for longer t_e, capturing the trial period's *dispersion effect*.

To maintain analytical tractability, we assume that

$$\epsilon_i - \xi_e(t_e) \sim \text{Uniform}\left[-(\nu + \delta t_e), \nu + \delta t_e\right], \tag{1.3}$$

where the constant $\nu \geq 0$ captures the variability embedded in ϵ_i while the constant $\delta > 0$ captures the dispersion effect embedded in $\xi_e(t_e)$. In particular, we also assume that $\bar{q} - \underline{q} \leq 2\delta$ so that the supports of the WTPs for the high-quality and low-quality products overlap even under the longest trial length $t_e = 1$ for any ν; otherwise, the problem becomes uninteresting.

To isolate the impact of competition between the incumbent and the entrant, we work in a *binary choice model* (e.g. Anderson et al. 1992, p. 34): the consumer has to choose to purchase one of the two products at the end of the trial period offered by the entrant. Namely, a consumer purchases the entrant's product if and only if $X_e(t_e, q_e, \widehat{q}_e) - p_e \geq X_i(q_i) - p_i$; otherwise, she purchases the incumbent's product.

2. Price Competition

In this section, we consider a hypothetical scenario, in which the entrant with publicly known quality q_e offers a trial period of length t_e and then engages in a simultaneous price competition with the incumbent. The incumbent's equilibrium price $p_i^\diamond(t_e, q_e)$ and the entrant's equilibrium price $p_e^\diamond(t_e, q_e)$ are jointly determined as the solution to the incumbent's profit-maximizing problem, which is given by

$$\Pi_i^\diamond(t_e, q_e) = \max_{p_i \geq 0} \; p_i \mathbb{P}\left[X_i(q_i) - p_i \geq X_e(t_e, q_e, q_e) - p_e^\diamond(t_e, q_e)\right], \tag{2.1}$$

and the entrant's profit-maximizing problem, which is given by

$$\Pi_e^\diamond(t_e, q_e) = \max_{p_e \geq 0} \; p_e \mathbb{P}\left[X_e(t_e, q_e, q_e) - p_e \geq X_i(q_i) - p_i^\diamond(t_e, q_e)\right]. \tag{2.2}$$

Following the standard solution procedure, we identify the Nash equilibrium of this subgame as our first result.

Proposition 1. *Denote $L := \nu + \delta t_e$. The equilibrium prices determined as solutions to (2.1) and (2.2) are given by*

$$(p_i^\diamond(t_e, q_e), p_e^\diamond(t_e, q_e)) = \begin{cases} (0, q_e - q_i - L), & \text{if} \quad 3L < q_e - q_i, \\ \left(\frac{1}{3}(q_i - q_e) + L, \frac{1}{3}(q_e - q_i) + L\right), & \text{if} \quad 3L \geq |q_e - q_i|, \\ (q_i - q_e - L, 0), & \text{if} \quad 3L < q_i - q_e. \end{cases} \tag{2.3}$$

The corresponding equilibrium expected profits are given by

$$(\Pi_i^\diamond(t_e, q_e), \Pi_e^\diamond(t_e, q_e)) = \begin{cases} (0, q_e - q_i - L), & \text{if} \quad 3L < q_e - q_i, \\ \left(\frac{1}{18L}(q_i - q_e + 3L)^2, \frac{1}{18L}(q_e - q_i + 3L)^2\right), & \text{if} \quad 3L \geq |q_e - q_i|, \\ (q_i - q_e - L, 0), & \text{if} \quad 3L < q_i - q_e. \end{cases} \tag{2.4}$$

As an immediate consequence of Proposition 1, the entrant's complete-information trial length $t_e^\circ(q_e)$ can be determined by maximizing its expected profit $\Pi_e^\diamond(t_e, q_e)$ prior to the price competition:

$$\Pi_e^\circ(q_e) := \max_{t_e \in [0,1]} \quad \Pi_e^\diamond(t_e, q_e). \tag{2.5}$$

The following proposition characterizes the complete-information outcome.

Proposition 2. *When the entrant's quality q_e is publicly known, the entrant offers a trial period of length*

$$t_e^\circ(q_e) = \begin{cases} 0, & \text{if} \quad q_e - q_i \geq 6(\nu + \delta) - 3\sqrt{(\nu + \delta)(\nu + 3\delta)}, \\ 1, & \text{if} \quad q_e - q_i < 6(\nu + \delta) - 3\sqrt{(\nu + \delta)(\nu + 3\delta)}, \end{cases} \tag{2.6}$$

and earns an expected profit of

$$\Pi_e^\circ(q_e) = \begin{cases} q_e - q_i - \nu, & \text{if} \quad q_e - q_i \geq 6(\nu+\delta) - 3\sqrt{(\nu+\delta)(\nu+3\delta)}, \\ \frac{[q_e - q_i + 3(\nu+\delta)]^2}{18(\nu+\delta)}, & \text{if} \quad -3(\nu+\delta) \leq q_e - q_i < 6(\nu+\delta) - 3\sqrt{(\nu+\delta)(\nu+3\delta)}, \\ 0, & \text{if} \quad q_e - q_i < -3(\nu+\delta). \end{cases} \tag{2.7}$$

In particular, the complete-information equilibrium profit $\Pi_e^\circ(q_e)$ *is monotonically increasing in* q_e*.*

Remark 1. The threshold $6(\nu+\delta) - 3\sqrt{(\nu+\delta)(\nu+3\delta)} \in (3\nu, 3(\nu+\delta))$.

As demonstrated by Proposition 2, when the entrant's product quality exceeds the incumbent's product quality by an obvious margin, the entrant's complete-information strategy is to offer no trial period. This result is consistent with the monopolistic case, whereby the firm is essentially competing with a no-purchase option. When the entrant has an obvious quality advantage over the incumbent, the entrant firm faces a very weak competitor and acts as if it were competing against a no-purchase option. As such, the trial period's dispersion effect is detrimental to the entrant's profitability, inducing the entrant not to offer any trial period.

When the entrant does not have such a sufficient quality advantage over or even has a lower quality level than that of the incumbent, Proposition 2 shows that the entrant has an incentive to use the longest possible trial length, which is distinct from the monopolistic complete-information case. In this case, the dispersion effect in fact benefits rather than hurts the entrant: it acts to soften the competition between the entrant and the incumbent by increasing the heterogeneity of the consumer's WTP for the entrant's product and helping reduce the consumer's price elasticity. In other words, by offering a longer trial period, the entrant can obfuscate the quality difference with the incumbent, whose product quality is comparable to or even higher than the entrant's quality. This observation is consistent with the results that have been verified in the literature using different consumer choice model assumptions (see e.g., Anderson and de Palma (1992) and Anderson et al. (1992), p. 230).

3. Signaling Stage

In this section, we turn to our focal case, where the entrant's product quality q_e is not publicly observable. After having characterized the incumbent's equilibrium pricing strategy (Proposition 1), we first work backwards and derive the entrant's expected profit function in the reduced form, which will facilitate the subsequent analysis of the entrant's signaling strategy.

Suppose that the entrant of quality q_e offers a trial period of length t_e and is subsequently being perceived as of quality type $\widehat{q}_e$ by the incumbent and the market. The incumbent will thus set its price at $p_i^\diamond(t_e, \widehat{q}_e)$, and the consumer's WTP for the entrant's product after the trial is given by (1.2). As a best response, the entrant maximizes its expected profit by optimizing its price accordingly:

$$\Pi_e\left(t_e, q_e, \widehat{q}_e\right) := \max_{p_e} \; p_e \mathbb{P}\left[X_e(t_e, q_e, \widehat{q}_e) - p_e \geq X_i(q_i) - p_i^{\diamond}\left(t_e, \widehat{q}_e\right)\right]. \tag{3.1}$$

The entrant's expected profit function in the reduced form above is characterized in the following lemma.

Lemma 1. *Let* $L := \nu + \delta t_e$. *Then,* $\Pi_e\left(t_e, q_e, q_e\right) = \Pi_e^{\diamond}\left(t_e, q_e\right)$ *for* $q_e \in \left\{\bar{q}, \underline{q}\right\}$.

$$\Pi_e\left(t_e, \underline{q}, \bar{q}\right) = \begin{cases} -\left(1 + \frac{\bar{q}-\underline{q}}{\delta}\right) L + \bar{q} - q_i + \frac{\nu}{\delta}(\bar{q} - \underline{q}), & \text{if} \quad \nu \leq L \leq \frac{\nu(\bar{q}-\underline{q}) + \delta(\bar{q}-q_i)}{3\delta + \bar{q} - \underline{q}}, \\ \left[\left(1 - \frac{\bar{q}-\underline{q}}{\delta}\right) L + \bar{q} - q_i + \frac{\nu}{\delta}(\bar{q} - \underline{q})\right]^2 / (8L), & \text{if} \quad \frac{\nu(\bar{q}-\underline{q}) + \delta(\bar{q}-q_i)}{3\delta + \bar{q} - \underline{q}} \leq L < \frac{\bar{q}-q_i}{3}, \\ \left[\left(2 - \frac{\bar{q}-\underline{q}}{\delta}\right) L + \frac{2}{3}\left(\bar{q} - q_i\right) + \frac{\nu}{\delta}(\bar{q} - \underline{q})\right]^2 / (8L), & \text{if} \quad \frac{\bar{q}-q_i}{3} \leq L \leq \nu + \delta; \end{cases} \tag{3.2}$$

$$\Pi_e\left(t_e, \bar{q}, \underline{q}\right) = \begin{cases} \left(\frac{\bar{q}-\underline{q}}{\delta} - 1\right) L + \underline{q} - q_i - \frac{\nu}{\delta}(\bar{q} - \underline{q}), & \text{if} \quad \nu \leq L < \frac{\underline{q}-q_i}{3}, \\ \frac{\bar{q}-\underline{q}}{\delta} L + \frac{2}{3}(\underline{q} - q_i) - \frac{\nu}{\delta}(\bar{q} - \underline{q}), & \text{if} \quad \frac{1}{3}|\underline{q} - q_i| \leq L \leq \frac{\frac{2}{3}\delta(\underline{q}-q_i) - \nu(\bar{q}-\underline{q})}{2\delta - (\bar{q}-\underline{q})}, \\ \left[\left(\frac{\bar{q}-\underline{q}}{\delta} + 2\right) L + \frac{2}{3}(\underline{q} - q_i) - \frac{\nu}{\delta}(\bar{q} - \underline{q})\right]^2 / (8L), & \text{if} \quad L \geq \max\left\{\frac{1}{3}|\underline{q} - q_i|, \frac{\frac{2}{3}\delta(\underline{q}-q_i) - \nu(\bar{q}-\underline{q})}{2\delta - (\bar{q}-\underline{q})}\right\}, \\ \left[(\bar{q} - \underline{q})\frac{L-\nu}{\delta}\right]^2 / (8L), & \text{if} \quad \nu \leq L < \frac{q_i-\underline{q}}{3}, \end{cases} \tag{3.3}$$

where, in particular,

$$\frac{1}{3}|\underline{q} - q_i| \leq \frac{\frac{2}{3}\delta(\underline{q} - q_i) - \nu(\bar{q} - \underline{q})}{2\delta - (\bar{q} - \underline{q})}, \quad \textit{if and only if} \quad \underline{q} - q_i \geq 3\nu. \tag{3.4}$$

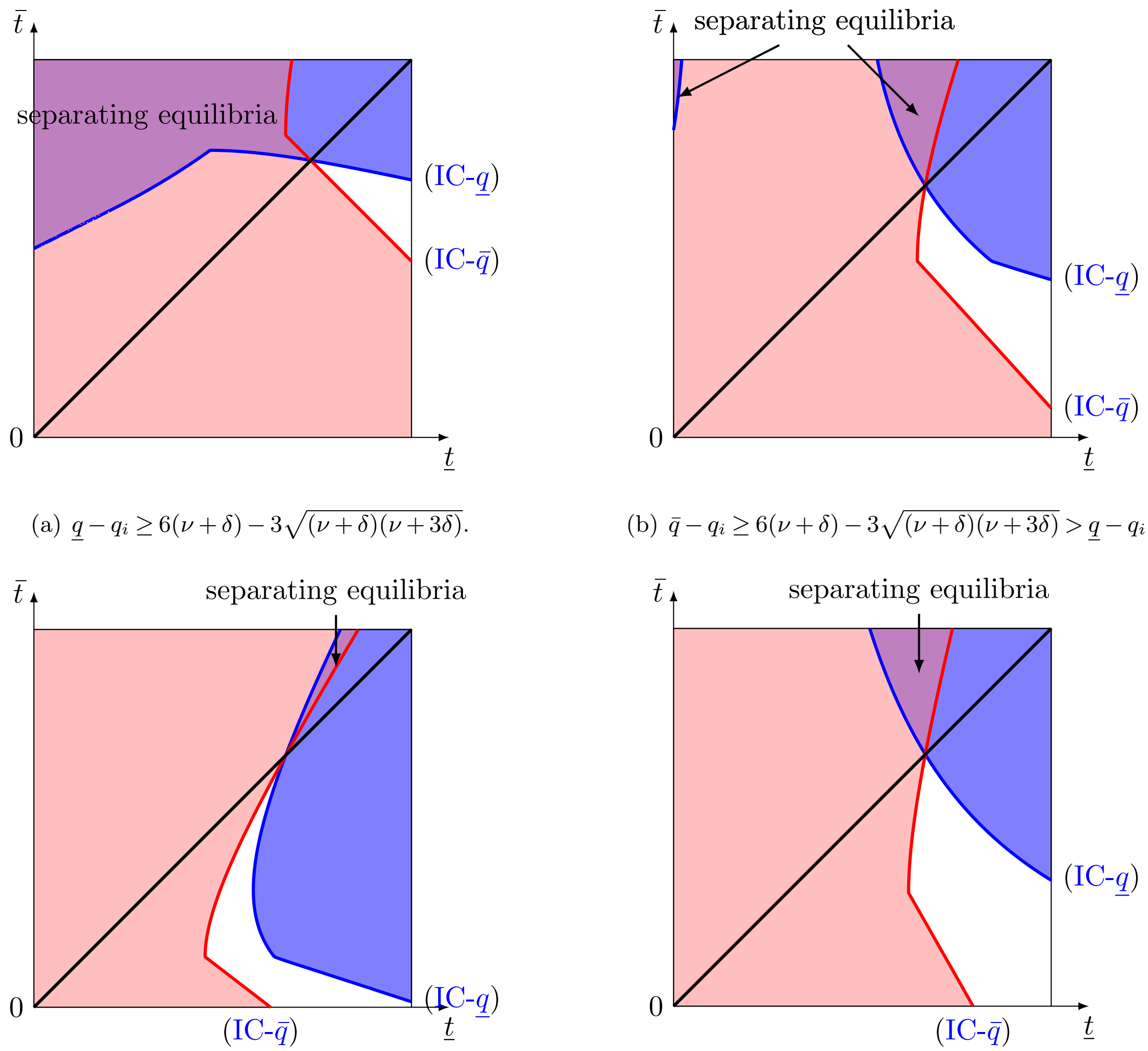
Having characterized the entrant's profit function in the reduced form, we can now examine the entrant's trial length strategies that separate its quality type. More specifically, let $\underline{t}$ and $\bar{t}$ denote the low-quality and the high-quality entrant's trial lengths, respectively. Then, $(\underline{t}, \bar{t})$ constitutes a *separating equilibrium* (i.e., $\underline{t} \neq \bar{t}$) if it satisfies the following incentive compatibility (IC) constraints:

$$\Pi_e\left(\bar{t}, \underline{q}, \bar{q}\right) \leq \Pi_e^{\diamond}(\underline{t}, \underline{q}), \tag{IC-$\underline{q}$}$$

$$\Pi_e\left(\underline{t}, \bar{q}, \underline{q}\right) \leq \Pi_e^{\diamond}(\bar{t}, \bar{q}). \tag{IC-$\bar{q}$}$$

The IC constraint (IC-$\underline{q}$) prevents the low-quality entrant from pretending to be its high-quality counterpart, and vice versa for (IC-$\bar{q}$). In the following proposition, we document the key property of any separating equilibrium, which is consistent with the qualitative result of the monopolistic case. That is, an entrant of higher product quality offers a longer trial period than a lower-quality entrant does.

Proposition 3. *For any separating* $(\underline{t}, \bar{t})$ *that satisfies* (IC-$\underline{q}$) *and* (IC-$\bar{q}$)*, we must have* $\underline{t} < \bar{t}$.



(a) $\underline{q}-q_i \geq 6(\nu+\delta)-3\sqrt{(\nu+\delta)(\nu+3\delta)}$.

(b) $\bar{q}-q_i \geq 6(\nu+\delta)-3\sqrt{(\nu+\delta)(\nu+3\delta)} > \underline{q}-q_i$.

(c) $6(\nu+\delta)-3\sqrt{(\nu+\delta)(\nu+3\delta)} > \bar{q}-q_i > \underline{q}-q_i \geq 0$.

(d) $6(\nu+\delta)-3\sqrt{(\nu+\delta)(\nu+3\delta)} > \bar{q}-q_i > 0 \geq \underline{q}-q_i$.

Figure 1 Separating equilibria for $\nu = 0.2, \delta = 1, q_i = 5$.

The significance of Proposition 3 lies in allowing us, without selecting a particular separating equilibrium, to claim that the set of *all* separating strategies defined by (IC-$\underline{q}$) and (IC-$\bar{q}$) must entail a longer trial period for a high-quality firm than for a low-quality firm. Figure 1 illustrates the mechanism underlying Proposition 3 for different parametric combinations. The blue-shaded area corresponds to the feasible trial strategies allowed by (IC-$\underline{q}$), while the red-shaded area corresponds to the feasible trial strategies allowed by (IC-$\bar{q}$). The intersection of these two areas is shaded in purple and represents all possible separating equilibria. The purple-shaded area being located above the diagonal line means that the trial length offered by the high-quality firm is longer than that offered by the low-quality firm. In particular, it is interesting to observe that the boundaries of the blue- and red-shaded areas intersect each other exactly on the diagonal of the trial strategy space. We also note that the low-quality firm's trial length is finite: while the high-quality firm can offer longest possible trial length, the low-quality firm never does so.

In the monopolistic case, we select and obtain an even sharper characterization of the separating equilibrium by selecting the *most efficient separating equilibrium* $(\underline{t}^\star, \bar{t}^\star)$: there does not exist another $(\underline{t}', \bar{t}')$ satisfying (IC-$\underline{q}$) and (IC-$\bar{q}$), under which the entrant of one quality type is strictly better off without the other quality type being worse off. In the most efficient separating equilibrium of the monopolistic market, the low-quality firm offers no trial period whereas the high-quality firm offers a trial period of positive length. As shown in Figures 1(a) and 1(b), the trial strategy of this nature is still feasible – the purple-shaded area intersects with the vertical axis (i.e., $\underline{t}$). Indeed, the parametric condition $\underline{q} - q_i \geq 6(\nu+\delta) - 3\sqrt{(\nu+\delta)(\nu+3\delta)}$ turns out to be a sufficient condition to select a trial strategy of this nature as the most efficient separating equilibrium.

First, the following proposition demonstrates that the low-quality firm always offers no trial period as in the complete-information strategy.

Proposition 4. *When $\underline{q} - q_i \geq 6(\nu+\delta) - 3\sqrt{(\nu+\delta)(\nu+3\delta)}$, in any separating equilibrium, the low-quality entrant ($\underline{q}$) always offers no trial period $\underline{t}_e^\star = 0$ and earns the complete-information expected profit $\Pi_e^\circ(\underline{q}) = \underline{q} - q_i - \nu$.*

When the entrant, even though its quality is low, has a significant quality advantage over the incumbent, the entrant, as in the monopolistic setting, has no incentive to offer a trial period, whose dispersion effect acts to obfuscate this quality advantage. Proposition 4 suggests that the high-quality firm's most efficient separating strategy is to offer a trial period of length $\bar{t}^\star$, which is determined as the solution to the following problem:

$$\bar{\pi}^\star := \max_{\bar{t}\geq 0} \; \Pi_e^\diamond(\bar{t}, \bar{q}), \quad \text{subject to } \; \Pi_e\left(\bar{t}, \underline{q}, \bar{q}\right) \leq \Pi_e^\circ(\underline{q}) \; \text{ and } \; \Pi_e\left(0, \bar{q}, \underline{q}\right) \leq \Pi_e^\diamond(\bar{t}, \bar{q}). \tag{3.5}$$

Proposition 5. *When $\underline{q} - q_i \geq 6(\nu+\delta) - 3\sqrt{(\nu+\delta)(\nu+3\delta)}$. in the most efficient separating equilibrium, the high-quality entrant ($\bar{q}$) offers a trial period of length $\bar{t}^\star \in (0,1)$, which uniquely satisfies $\Pi_e\left(\bar{t}^\star, \underline{q}, \bar{q}\right) = \Pi_e^\circ(\underline{q})$, and earns an expected profit of $\bar{\pi}^\star = \Pi_e^\diamond(\bar{t}^\star, \bar{q})$.*

The result in Proposition 5, together with that in Proposition 4, is consistent with our finding in the monopolistic setting in that the high-quality entrant needs to offer a long enough trial period in order to establish its superior quality image in the marketplace. When the entrant has a significant quality advantage over the incumbent (i.e., $\bar{q} - q_i > \underline{q} - q_i \geq 6(\nu+\delta) - 3\sqrt{(\nu+\delta)(\nu+3\delta)}$), the entrant's trial strategy is primarily driven by its need to signal its quality type, which dominates the need to compete with the incumbent. As such, all the qualitative findings in the monopolistic setting are carried over to the competitive setting.

For parametric conditions other than the one in Proposition 5, the set of separating equilibria may entail positive trial lengths by the low-quality entrant, i.e., $\underline{t} > 0$, which is illustrated by Figure 1. This suggests that the low-quality entrant offers a trial period of positive trial length. Nonetheless, according to Proposition 3, the high-quality entrant offers an even longer trial period to distinguish its superior quality from that of the low-quality entrant. Therefore, the qualitative characterization of the equilibrium trial strategies made for the monopolistic case still holds for such parametric conditions.

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Appendix

Proof of Proposition 1. By maximizing its profit below for a given entrant's price p_e,

$$p_i\mathbb{P}\left[X_i(q_i)-p_i \geq X_e(t_e,q_e,q_e)-p_e\right] = \begin{cases} 0, & \text{if} \quad q_i-q_e+p_e-p_i \leq -L, \\ \frac{1}{2L}p_i\left(q_i-q_e+p_e-p_i+L\right), & \text{if} \quad -L \leq q_i-q_e+p_e-p_i \leq L, \\ p_i, & \text{if} \quad q_i-q_e+p_e-p_i > L, \end{cases} \tag{3.6}$$

the incumbent sets the following price as the best response:

$$p_i^{\diamond}\left(p_e \mid t_e,q_e\right) = \begin{cases} \frac{1}{2}(q_i-q_e+p_e+L), & \text{if} \quad p_e \leq 3L+q_e-q_i, \\ q_i-q_e+p_e-L, & \text{if} \quad p_e > 3L+q_e-q_i. \end{cases} \tag{3.7}$$

Similarly, by maximizing its profit below for a given incumbent price p_i,

$$p_e\mathbb{P}\left[X_e(t_e,q_e,q_e)-p_e \geq X_i(q_i)-p_i\right] = \begin{cases} 0, & \text{if} \quad q_e-q_i+p_i-p_e \leq -L, \\ \frac{1}{2L}p_e\left(q_e-q_i+p_i-p_e+L\right), & \text{if} \quad -L \leq q_e-q_i+p_i-p_e \leq L, \\ p_e, & \text{if} \quad q_e-q_i+p_i-p_e > L, \end{cases} \tag{3.8}$$

the entrant sets the following price as the best response:

$$p_e^{\diamond}\left(p_i \mid t_e,q_e\right) = \begin{cases} \frac{1}{2}(q_e-q_i+p_i+L), & \text{if} \quad p_i \leq 3L+q_i-q_e, \\ q_e-q_i+p_i-L, & \text{if} \quad p_i > 3L+q_i-q_e. \end{cases} \tag{3.9}$$

The intersection of (3.7) and (3.9) yields (2.3) as the equilibrium prices, which we substitute into (3.6) and (3.8) to obtain (2.4). □

Proof of Proposition 2. Recall that $L=\nu+\delta t_e\in[\nu,\nu+\delta]$. Thus, optimization over t_e is equivalent to optimization over L. We consider the following cases:

- If $q_e-q_i\ge 3(\nu+\delta)$, then $3L<q_e-q_i$ for all L and, by (2.4), $\Pi_e^\diamond(t_e,q_e)=q_e-q_i-L$, which is maximized by $L=\nu$ or equivalently $t_e=0$. Therefore, in this case, $\Pi_e^\circ(q_e)=q_e-q_i-\nu$.
- If $3\nu\le q_e-q_i\le 3(\nu+\delta)$, then by (2.4), $\Pi_e^\diamond(t_e,q_e)=q_e-q_i-L$ for $\nu\le L<\frac{1}{3}(q_e-q_i)$, which reaches its maximum $q_e-q_i-\nu$ at $L=\nu$; while $\Pi_e^\diamond(t_e,q_e)=\frac{1}{18L}(q_e-q_i+3L)^2=\frac{1}{18}\left[9L+\frac{(q_e-q_i)^2}{L}+6(q_e-q_i)\right]$ for $\frac{1}{3}(q_e-q_i)\le L\le\nu+\delta$, which reaches its maximum $\frac{1}{18(\nu+\delta)}[q_e-q_i+3(\nu+\delta)]^2$ at $L=\nu+\delta$. Straightforward calculation reveals

$$\begin{aligned}&\frac{1}{18(\nu+\delta)}[q_e-q_i+3(\nu+\delta)]^2-(q_e-q_i-\nu)\\=&\frac{1}{18(\nu+\delta)}\left[(q_e-q_i)^2-12(\nu+\delta)(q_e-q_i)+9(\nu+\delta)^2+18\nu(\nu+\delta)\right]\\=&\frac{1}{18(\nu+\delta)}\left[(q_e-q_i-6(\nu+\delta))^2-9(\nu+\delta)(\nu+3\delta)\right]\end{aligned}$$

which is positive if and only if $q_e-q_i<6(\nu+\delta)-3\sqrt{(\nu+\delta)(\nu+3\delta)}$. In particular, it is straightforward to verify that $3\nu<6(\nu+\delta)-3\sqrt{(\nu+\delta)(\nu+3\delta)}<3(\nu+\delta)$. Therefore, we have $\Pi_e^\diamond(t_e,q_e)$ is is maximized by $L=\nu+\delta$ or equivalently $t_e=1$ when $q_e-q_i<6(\nu+\delta)-3\sqrt{(\nu+\delta)(\nu+3\delta)}$, and otherwise is maximized by $L=\nu$ or equivalently $t_e=0$.
- If $-3(\nu+\delta)\le q_e-q_i\le 3\nu$, we must have $3L\ge|q_e-q_i|$ and hence, by (2.4), $\Pi_e^\diamond(t_e,q_e)=\frac{1}{18L}(q_e-q_i+3L)^2$ for all $L\ge\frac{1}{3}|q_e-q_i|$. in particular, it is increasing in $L\ge\frac{1}{3}|q_e-q_i|$ and reaches its maximum $\frac{[q_e-q_i+3(\nu+\delta)]^2}{18(\nu+\delta)}$ by $L=\nu+\delta$ or equivalently $t_e=1$.
- If $q_e-q_i<-3(\nu+\delta)$, then we must have $3L<q_i-q_e$ and hence $\Pi_e^\diamond(t_e,q_e)\equiv 0$. $\square$

Proof of Lemma 1. By definition,

$$\begin{aligned}&p_e\mathbb{P}\left[X_e(t_e,q_e,\widehat{q}_e)-p_e\ge X_i(q_i)-p_i^\diamond(t_e,\widehat{q}_e)\right]\\=&p_e\mathbb{P}\left[\epsilon_i-\xi_e(t_e)\le\mu(t_e,q_e,\widehat{q}_e)-q_i-p_e+p_i^\diamond(t_e,\widehat{q}_e)\right]\\=&\begin{cases}0, & \text{if}\quad \mu(t_e,q_e,\widehat{q}_e)-q_i-p_e+p_i^\diamond(t_e,\widehat{q}_e)<-L,\\ p_e\left[\mu(t_e,q_e,\widehat{q}_e)-q_i-p_e+p_i^\diamond(t_e,\widehat{q}_e)\right]/(2L), & \text{if}\quad -L\le\mu(t_e,q_e,\widehat{q}_e)-q_i-p_e+p_i^\diamond(t_e,\widehat{q}_e)\le L,\\ p_e, & \text{if}\quad \mu(t_e,q_e,\widehat{q}_e)-q_i-p_e+p_i^\diamond(t_e,\widehat{q}_e)>L,\end{cases}\end{aligned}$$

which is maximized by the following entrant's price

$$p_e(t_e,q_e,\widehat{q}_e)=\begin{cases}0, & \text{if}\quad \mu(t_e,q_e,\widehat{q}_e)-q_i+p_i^\diamond(t_e,\widehat{q}_e)\le -L,\\ \left[\mu(t_e,q_e,\widehat{q}_e)-q_i+p_i^\diamond(t_e,\widehat{q}_e)+L\right]/2, & \text{if}\quad -L\le\mu(t_e,q_e,\widehat{q}_e)-q_i+p_i^\diamond(t_e,\widehat{q}_e)\le 3L,\\ \mu(t_e,q_e,\widehat{q}_e)-q_i+p_i^\diamond(t_e,\widehat{q}_e)-L, & \text{if}\quad \mu(t_e,q_e,\widehat{q}_e)-q_i+p_i^\diamond(t_e,\widehat{q}_e)\ge 3L,\end{cases}$$

and reaches the maximum value of

$$\Pi_e(t_e,q_e,\widehat{q}_e)=\begin{cases}0, & \text{if}\quad \mu(t_e,q_e,\widehat{q}_e)-q_i+p_i^\diamond(t_e,\widehat{q}_e)\le -L,\\ \left[\mu(t_e,q_e,\widehat{q}_e)-q_i+p_i^\diamond(t_e,\widehat{q}_e)+L\right]^2/(8L), & \text{if}\quad -L\le\mu(t_e,q_e,\widehat{q}_e)-q_i+p_i^\diamond(t_e,\widehat{q}_e)\le 3L,\\ \mu(t_e,q_e,\widehat{q}_e)-q_i+p_i^\diamond(t_e,\widehat{q}_e)-L, & \text{if}\quad \mu(t_e,q_e,\widehat{q}_e)-q_i+p_i^\diamond(t_e,\widehat{q}_e)\ge 3L.\end{cases}\tag{3.10}$$

By (2.3), it is straightforward to verify that

$$\mu(t_e,q_e,\widehat{q}_e)-q_i+p_i^\diamond(t_e,\widehat{q}_e)=(q_e-\widehat{q}_e)\frac{L-\nu}{\delta}+\begin{cases}\widehat{q}_e-q_i, & \text{if} \quad 3L<\widehat{q}_e-q_i,\\ \frac{2}{3}(\widehat{q}_e-q_i)+L, & \text{if} \quad 3L\geq|\widehat{q}_e-q_i|,\\ -L, & \text{if} \quad 3L<q_i-\widehat{q}_e,\end{cases} \tag{3.11}$$

and subsequently $\Pi_e(t_e,q_e,q_e)=\Pi_e^\diamond(t_e,q_e)$ by letting $\widehat{q}_e=q_e$.

- If $q_e=\underline{q}$ and $\widehat{q}_e=\bar{q}$, there are two possible scenarios:

1. For $3L<\bar{q}-q_i$, we must have $\bar{q}-q_i>3\nu$ and, according to (3.11),

$$\mu(t_e,\underline{q},\bar{q})-q_i+p_i^\diamond(t_e,\bar{q})=(\underline{q}-\bar{q})\frac{L-\nu}{\delta}+\bar{q}-q_i\geq-2\delta\frac{L-\nu}{\delta}+3L\geq L,$$

where we use the assumption that $\bar{q}-\underline{q}\leq 2\delta$. On the other hand, we can easily verify that

$$\mu(t_e,\underline{q},\bar{q})-q_i+p_i^\diamond(t_e,\bar{q})=(\underline{q}-\bar{q})\frac{L-\nu}{\delta}+\bar{q}-q_i\leq 3L, \text{ if and only if } L\geq\frac{\nu(\bar{q}-\underline{q})+\delta(\bar{q}-q_i)}{3\delta+\bar{q}-\underline{q}}.$$

Therefore, (3.10) suggests

$$\Pi_e(t_e,\underline{q},\bar{q})=\begin{cases}\left[\left(1-\frac{\bar{q}-\underline{q}}{\delta}\right)L+\bar{q}-q_i+\frac{\nu}{\delta}(\bar{q}-\underline{q})\right]^2/(8L), & \text{if} \quad \frac{\nu(\bar{q}-\underline{q})+\delta(\bar{q}-q_i)}{3\delta+\bar{q}-\underline{q}}\leq L<\frac{\bar{q}-q_i}{3},\\ -\left(1+\frac{\bar{q}-\underline{q}}{\delta}\right)L+\bar{q}-q_i+\frac{\nu}{\delta}(\bar{q}-\underline{q}), & \text{if} \quad \nu\leq L\leq\frac{\nu(\bar{q}-\underline{q})+\delta(\bar{q}-q_i)}{3\delta+\bar{q}-\underline{q}}.\end{cases} \tag{3.12}$$

2. For $3L\geq\bar{q}-q_i>0$, we must have $\bar{q}-q_i\leq 3(\nu+\delta)$ and, according to (3.11),

$$\mu(t_e,\underline{q},\bar{q})-q_i+p_i^\diamond(t_e,\bar{q})=\left(1-\frac{\bar{q}-\underline{q}}{\delta}\right)L+\frac{2}{3}(\bar{q}-q_i)+\frac{\nu}{\delta}(\bar{q}-\underline{q})=-(\bar{q}-\underline{q})t_e+\frac{2}{3}(\bar{q}-q_i)+L\leq 3L.$$

At the same time, the assumption that $0<\bar{q}-\underline{q}\leq 2\delta$ suggests that

$$\mu(t_e,\underline{q},\bar{q})-q_i+p_i^\diamond(t_e,\bar{q})=\left(1-\frac{\bar{q}-\underline{q}}{\delta}\right)L+\frac{2}{3}(\bar{q}-q_i)+\frac{\nu}{\delta}(\bar{q}-\underline{q})\geq -L.$$

Therefore, (3.10) suggests

$$\Pi_e(t_e,\underline{q},\bar{q})=\left[\left(2-\frac{\bar{q}-\underline{q}}{\delta}\right)L+\frac{2}{3}(\bar{q}-q_i)+\frac{\nu}{\delta}(\bar{q}-\underline{q})\right]^2/(8L), \quad \text{for } \frac{\bar{q}-q_i}{3}\leq L\leq\nu+\delta. \tag{3.13}$$

In summary, (3.2) immediately follows from (3.12) and (3.13).

- If $q_e=\bar{q}$ and $\widehat{q}_e=\underline{q}$, there are three possible scenarios:

1. For $3L<\underline{q}-q_i$, we must have $\underline{q}-q_i>3\nu$ and, according to (3.11),

$$\mu(t_e,\bar{q},\underline{q})-q_i+p_i^\diamond(t_e,\underline{q})=(\bar{q}-\underline{q})\frac{L-\nu}{\delta}+\underline{q}-q_i>3L.$$

Therefore, (3.10) suggests

$$\Pi_e(t_e,\bar{q},\underline{q})=\left(\frac{\bar{q}-\underline{q}}{\delta}-1\right)L+\underline{q}-q_i-\frac{\nu}{\delta}(\bar{q}-\underline{q}), \quad \text{for } \nu\leq L<\frac{\underline{q}-q_i}{3}. \tag{3.14}$$

2. For $3L \geq |\underline{q} - q_i|$, we must have $|\underline{q} - q_i| \leq 3(\nu + \delta)$ and, according to (3.11),

$$\mu(t_e, \bar{q}, \underline{q}) - q_i + p_i^\diamond(t_e, \underline{q}) = \left(\frac{\bar{q} - \underline{q}}{\delta} + 1\right) L + \frac{2}{3}(\underline{q} - q_i) - \frac{\nu}{\delta}(\bar{q} - \underline{q}) = (\bar{q} - \underline{q})t_e + L + \frac{2}{3}(\underline{q} - q_i) \geq L - 2L = -L.$$

On the other hand, we can easily verify that

$$\mu(t_e, \bar{q}, \underline{q}) - q_i + p_i^\diamond(t_e, \underline{q}) = \left(\frac{\bar{q} - \underline{q}}{\delta} + 1\right) L + \frac{2}{3}(\underline{q} - q_i) - \frac{\nu}{\delta}(\bar{q} - \underline{q}) \geq 3L,$$

$$\text{if and only if } L \leq \frac{\frac{2}{3}\delta(\underline{q} - q_i) - \nu(\bar{q} - \underline{q})}{2\delta - (\bar{q} - \underline{q})}.$$

Therefore, (3.10) suggests

$$\Pi_e(t_e, \bar{q}, \underline{q}) = \begin{cases} \frac{\bar{q} - \underline{q}}{\delta} L + \frac{2}{3}(\underline{q} - q_i) - \frac{\nu}{\delta}(\bar{q} - \underline{q}), & \text{if } \frac{1}{3}|\underline{q} - q_i| \leq L \leq \frac{\frac{2}{3}\delta(\underline{q} - q_i) - \nu(\bar{q} - \underline{q})}{2\delta - (\bar{q} - \underline{q})}, \\ \left[\left(\frac{\bar{q} - \underline{q}}{\delta} + 2\right) L + \frac{2}{3}(\underline{q} - q_i) - \frac{\nu}{\delta}(\bar{q} - \underline{q})\right]^2 / (8L), & \text{if } L \geq \max\left\{\frac{1}{3}|\underline{q} - q_i|, \frac{\frac{2}{3}\delta(\underline{q} - q_i) - \nu(\bar{q} - \underline{q})}{2\delta - (\bar{q} - \underline{q})}\right\}. \end{cases} \tag{3.15}$$

3. For $3L < q_i - \underline{q}$, we must have $q_i - \underline{q} > 3\nu$ and, according to (3.11),

$$\mu(t_e, \bar{q}, \underline{q}) - q_i + p_i^\diamond(t_e, \underline{q}) = (\bar{q} - \underline{q}) \frac{L - \nu}{\delta} - L \geq -L.$$

On the other hand, using the assumption that $\bar{q} - \underline{q} \leq 2\delta$, we can easily verify that

$$\mu(t_e, \bar{q}, \underline{q}) - q_i + p_i^\diamond(t_e, \underline{q}) = (\bar{q} - \underline{q}) \frac{L - \nu}{\delta} - L \leq L < 3L.$$

Therefore, (3.10) suggests

$$\Pi_e(t_e, \bar{q}, \underline{q}) = \left[(\bar{q} - \underline{q}) \frac{L - \nu}{\delta}\right]^2 / (8L), \quad \text{for } \nu \leq L < \frac{q_i - \underline{q}}{3}. \tag{3.16}$$

In summary, (3.3) immediately follows from (3.14), (3.15) and (3.16). □

Proof of Proposition 3. We will only show the case where $\left|\underline{q} - q_i\right| < 3\nu$ and $0 < \bar{q} - q_i < 3\nu$; all the other cases follow from similar argument. In this case, (IC-$\underline{q}$) and (IC-$\bar{q}$) reduce to

$$\frac{1}{8\bar{L}} \left[\left(2 - \frac{\bar{q} - \underline{q}}{\delta}\right) \bar{L} + \frac{2}{3}(\bar{q} - q_i) + \frac{\nu}{\delta}(\bar{q} - \underline{q})\right]^2 \leq \frac{1}{18\underline{L}}(\underline{q} - q_i + 3\underline{L})^2, \tag{3.17}$$

$$\frac{1}{8\underline{L}} \left[\left(\frac{\bar{q} - \underline{q}}{\delta} + 2\right) \underline{L} + \frac{2}{3}(\underline{q} - q_i) - \frac{\nu}{\delta}(\bar{q} - \underline{q})\right]^2 \leq \frac{1}{18\bar{L}}(\bar{q} - q_i + 3\bar{L})^2, \tag{3.18}$$

where $\underline{L} := \nu + \delta\underline{t}$ and $\bar{L} := \nu + \delta\bar{t}$. It is straightforward to verify that $\underline{L} = \bar{L} = \nu + \frac{2}{3}\delta$ binds both (3.17) and (3.17).

For any $\bar{L}$, let $\underline{L}^* = \underline{L}^*(\bar{L})$ and $\underline{L}_* = \underline{L}_*(\bar{L})$ be such that $(\underline{L}^*, \bar{L})$ binds (3.17) and $(\underline{L}_*, \bar{L})$ binds (3.18). Namely,

$$\frac{9}{4\bar{L}} \left[\left(2 - \frac{\bar{q} - \underline{q}}{\delta}\right) \bar{L} + \frac{2}{3}(\bar{q} - q_i) + \frac{\nu}{\delta}(\bar{q} - \underline{q})\right]^2 = \frac{1}{\underline{L}^*}(\underline{q} - q_i + 3\underline{L}^*)^2, \tag{3.19}$$

$$\frac{9}{4\underline{L}_*}\left[\left(\frac{\bar{q}-\underline{q}}{\delta}+2\right)\underline{L}_*+\frac{2}{3}(\underline{q}-q_i)-\frac{\nu}{\delta}(\bar{q}-\underline{q})\right]^2=\frac{1}{\bar{L}}(\bar{q}-q_i+3\bar{L})^2. \tag{3.20}$$

Therefore, we must have

$$\underline{L}_*\le\bar{L} \text{ and } \underline{L}^*\le\bar{L}, \text{ if and only if } \bar{L}\ge\nu+\frac{2}{3}\delta, \tag{3.21}$$

with strictly inequality if $\bar{L}>\nu+\frac{2}{3}\delta$.

Since

$$\frac{1}{\underline{L}}(\underline{q}-q_i+3\underline{L})^2=\left(\frac{\underline{q}-q_i+3\underline{L}}{\sqrt{\underline{L}}}\right)^2 \quad \text{and}$$
$$\frac{9}{4\underline{L}}\left[\left(\frac{\bar{q}-\underline{q}}{\delta}+2\right)\underline{L}+\frac{2}{3}(\underline{q}-q_i)-\frac{\nu}{\delta}(\bar{q}-\underline{q})\right]^2=\left[\frac{\underline{q}-q_i+3\underline{L}+\frac{3}{2}\frac{\bar{q}-\underline{q}}{\delta}(\underline{L}-\nu)}{\sqrt{\underline{L}}}\right]^2$$

are both increasing in $\underline{L}$, the incentive feasible pairs $(\underline{L},\bar{L})$ must satisfies

$$\underline{L}^*\le\underline{L}\le\underline{L}_*, \quad \text{for each } \bar{L}. \tag{3.22}$$

We claim that $\underline{L}^*>\underline{L}_*$ if and only if $\bar{L}<\nu+\frac{2}{3}\delta$, thus suggesting that (3.22) is possible only for $\bar{L}\ge\nu+\frac{2}{3}\delta$ and hence that $\underline{L}\le\underline{L}_*\le\bar{L}$ with strictly inequality if $\bar{L}>\nu+\frac{2}{3}\delta$. As such, we obtain our result $\underline{t}<\bar{t}$.

To prove our claim, we note that, for $\bar{L}<\nu+\frac{2}{3}\delta$,

$$\begin{aligned}
\frac{1}{\sqrt{\underline{L}^*}}(\underline{q}-q_i+3\underline{L}^*)&=\frac{1}{\sqrt{\bar{L}}}\left[\bar{q}-q_i+3\bar{L}+\frac{3}{2}\frac{\bar{q}-\underline{q}}{\delta}(\bar{L}-\nu)\right] \quad (\text{by } (3.19))\\
&=\frac{1}{\sqrt{\underline{L}_*}}\left[\underline{q}-q_i+3\underline{L}_*+\frac{3}{2}\frac{\bar{q}-\underline{q}}{\delta}(\underline{L}_*-\nu)\right]-\frac{3}{2}\frac{\bar{q}-\underline{q}}{\delta}\frac{\bar{L}-\nu}{\sqrt{\bar{L}}} \quad (\text{by } (3.20))\\
&=\frac{1}{\sqrt{\underline{L}_*}}\left(\underline{q}-q_i+3\underline{L}_*\right)+\frac{3}{2}\frac{\bar{q}-\underline{q}}{\delta}\left(\frac{\underline{L}_*-\nu}{\sqrt{\underline{L}_*}}-\frac{\bar{L}-\nu}{\sqrt{\bar{L}}}\right)\\
&>\frac{1}{\sqrt{\underline{L}_*}}\left(\underline{q}-q_i+3\underline{L}_*\right),
\end{aligned}$$

where the last inequality following from the fact that $\underline{L}_*>\bar{L}$ for $\bar{L}<\nu+\frac{2}{3}\delta$ according to (3.21). The above inequality immediately implies that $\underline{L}^*>\underline{L}_*$ because $\frac{1}{\sqrt{\underline{L}}}(\underline{q}-q_i+3\underline{L})$ is an increasing function of $\underline{L}$. □

Proof of Proposition 4. Suppose that the low-quality entrant ($\underline{q}$) offers a trial period of positive length $\underline{t}>0$ or equivalently $L>\nu$ in the separating equilibrium, in which its expected profit would be $\Pi_e(\underline{t},\underline{q},\underline{q})=\Pi_e^\diamond(\underline{t},\underline{q})$. We now demonstrate that this entrant will be better off by deviating to $\underline{t}_e^\circ(\underline{q})=0$. By doing so, suppose that the entrant is recognized as $\widehat{q}_e\ge\underline{q}$ and thus earns expected profit of $\Pi_e(0,\underline{q},\widehat{q}_e)$:

- If $\widehat{q}_e = \underline{q}$, then we immediately have $\Pi_e(0,\underline{q},\underline{q}) = \Pi_e^\circ(\underline{q}) > \Pi_e^\diamond(\underline{t},\underline{q}) = \Pi_e(\underline{t},\underline{q},\underline{q})$ as $\underline{t}_e^\circ(\underline{q}) = 0$ is the complete-information trial length.
- If $\widehat{q}_e = \bar{q}$, then by (3.2),

$$\Pi_e(0,\underline{q},\bar{q}) = -\left(1+\frac{\bar{q}-\underline{q}}{\delta}\right)\nu + \bar{q}-q_i+\frac{\nu}{\delta}(\bar{q}-\underline{q}) = \bar{q}-q_i-\nu > \bar{q}-q_i-\nu = \Pi_e^\circ(\underline{q}) > \Pi_e^\diamond(\underline{t},\underline{q}) = \Pi_e(\underline{t},\underline{q},\underline{q}),$$

as $\bar{q}-q_i > \underline{q}-q_i \geq 6(\nu+\delta) - 3\sqrt{(\nu+\delta)(\nu+3\delta)} > 3\nu$ (by Remark 1) suggests that $\nu \leq \frac{\nu(\bar{q}-\underline{q})+\delta(\bar{q}-q_i)}{3\delta+\bar{q}-\underline{q}}$.

Therefore, in any separating equilibrium, the entrant of quality $\underline{q}$ must offer zero trial length as in the complete-information strategy and earns the complete-information profit level. □

Proof of Proposition 5. In this proof, we denote $\bar{L} = \nu + \delta\bar{t}$. To solve (3.5), we first ignore the second constraint, which by (3.3) can be written as

$$\Pi_e^\diamond(\bar{t},\bar{q}) \geq \Pi_e(0,\bar{q},\underline{q}) = \underline{q}-q_i-\nu = \Pi_e^\circ(\underline{q}). \tag{3.23}$$

We examine the feasible set specified by the first constraint $\Pi_e\left(\bar{t},\underline{q},\bar{q}\right) \leq \Pi_e^\circ(\underline{q})$. We first note that, for $\bar{q}-q_i > \underline{q}-q_i \geq 6(\nu+\delta) - 3\sqrt{(\nu+\delta)(\nu+3\delta)} > 3\nu$ (by Remark 1), (2.4) implies that

$$\Pi_e^\diamond(\bar{t},\bar{q}) = \begin{cases} \bar{q}-q_i-\bar{L}, & \text{if } \bar{L} < \frac{1}{3}(\bar{q}-q_i), \\ \frac{1}{2\bar{L}}\left[\frac{1}{3}(\bar{q}-q_i)+\bar{L}\right]^2, & \text{if } \bar{L} \geq \frac{1}{3}(\bar{q}-q_i), \end{cases} \tag{3.24}$$

which is decreasing in $\bar{L} < \frac{1}{3}(\bar{q}-q_i)$ and increasing in $\bar{L} \geq \frac{1}{3}(\bar{q}-q_i)$.

According to (3.2), we examine $\Pi_e\left(\bar{t},\underline{q},\bar{q}\right)$ as follows.

- When $\nu \leq \bar{L} \leq \frac{\nu(\bar{q}-\underline{q})+\delta(\bar{q}-q_i)}{3\delta+\bar{q}-\underline{q}} \leq \frac{1}{3}(\bar{q}-q_i)$, we have

$$\Pi_e\left(\bar{t},\underline{q},\bar{q}\right) = -\left(1+\frac{\bar{q}-\underline{q}}{\delta}\right)\bar{L} + \bar{q}-q_i+\frac{\nu}{\delta}(\bar{q}-\underline{q}) \leq \bar{q}-q_i-\bar{L} = \Pi_e^\diamond(\bar{t},\bar{q}), \tag{3.25}$$

and $\Pi_e\left(\bar{t},\underline{q},\bar{q}\right)$ is obviously decreasing in $\bar{L}$ with

$$\Pi_e\left(0,\underline{q},\bar{q}\right) = \bar{q}-q_i-\nu > \underline{q}-q_i-\nu = \Pi_e^\circ(\underline{q}). \tag{3.26}$$

In particular, if $\nu+\delta \leq \frac{\nu(\bar{q}-\underline{q})+\delta(\bar{q}-q_i)}{3\delta+\bar{q}-\underline{q}}$ or equivalently $\underline{q}-q_i \geq 3(\nu+\delta)$, we further have

$$\Pi_e\left(1,\underline{q},\bar{q}\right) = \underline{q}-q_i-(\nu+\delta) < \underline{q}-q_i-\nu = \Pi_e^\circ(\underline{q}),$$

which immediately implies that there uniquely exists $\bar{t}^\star \in (0,1)$ such that $\Pi(\bar{t},\underline{q},\bar{q}) \leq \Pi_e^\circ(\underline{q})$, if and only if $\bar{t} \geq \bar{t}^\star$. Furthermore, by (3.25), $\Pi_e^\diamond(\bar{t}^\star,\bar{q}) \geq \Pi_e\left(\bar{t}^\star,\underline{q},\bar{q}\right) = \Pi_e^\circ(\underline{q})$, i.e., the ignored constraint holds. Therefore, as $\Pi_e^\diamond(\bar{t},\bar{q})$ is decreasing in $\bar{t}$, we must have $\bar{t}^\star$ as the solution to (3.5).

- When $\bar{L} \in \left[\frac{\nu(\bar{q}-\underline{q})+\delta(\bar{q}-q_i)}{3\delta+\bar{q}-\underline{q}}, \frac{\bar{q}-q_i}{3}\right]$, we have

$$\Pi_e\left(\bar{t},\underline{q},\bar{q}\right) = \left[\left(1-\frac{\bar{q}-\underline{q}}{\delta}\right)\bar{L}+\bar{q}-q_i+\frac{\nu}{\delta}(\bar{q}-\underline{q})\right]^2/(8\bar{L})$$
$$=\frac{1}{8}\left[\left(1-\frac{\bar{q}-\underline{q}}{\delta}\right)^2\bar{L}+\frac{\left(\bar{q}-q_i+\nu/\delta(\bar{q}-\underline{q})\right)^2}{\bar{L}}+2\left(1-\frac{\bar{q}-\underline{q}}{\delta}\right)\left(\bar{q}-q_i+\frac{\nu}{\delta}(\bar{q}-\underline{q})\right)\right],$$

which is decreasing in $\bar{L} \le \frac{\bar{q}-q_i}{3} \le \frac{\bar{q}-q_i+\nu/\delta(\bar{q}-\underline{q})}{\left|1-\frac{\bar{q}-\underline{q}}{\delta}\right|}$ by noticing that $\left|1-\frac{\bar{q}-\underline{q}}{\delta}\right| \le 1$ implied by the assumption $0<\bar{q}-\underline{q}\le 2\delta$.

If $\frac{\nu(\bar{q}-\underline{q})+\delta(\bar{q}-q_i)}{3\delta+\bar{q}-\underline{q}} \le \nu+\delta \le \frac{\bar{q}-q_i}{3}$ or equivalently $\underline{q}-q_i \le 3(\nu+\delta) \le \bar{q}-q_i$, straightforward verification reveals that

$$\Pi_e\left(1,\underline{q},\bar{q}\right)-\Pi_e^{\circ}(\underline{q}) = \frac{1}{8(\nu+\delta)}\left(\underline{q}-q_i+\nu+\delta\right)^2-\left(\underline{q}-q_i-\nu\right)$$
$$=\frac{1}{8(\nu+\delta)}\left\{\left[3(\nu+\delta)-(\underline{q}-q_i)\right]^2-8\delta(\nu+\delta)\right\}$$
$$\le\frac{1}{8(\nu+\delta)}\left\{\left[3(\nu+\delta)-6(\nu+\delta)+3\sqrt{(\nu+\delta)(\nu+3\delta)}\right]^2-8\delta(\nu+\delta)\right\}$$
$$=\frac{1}{4}\left[9\nu+14\delta-9\sqrt{(\nu+\delta)(\nu+3\delta)}\right]<0.$$

Therefore, there uniquely exists a $\bar{t}^\star \in (0,1)$ such that $\Pi(\bar{t},\underline{q},\bar{q}) \le \Pi_e^{\circ}(\underline{q})$, if and only if $\bar{t} \ge \bar{t}^\star$.

We further claim that $\Pi_e\left(\bar{t},\underline{q},\bar{q}\right) \le \Pi_e^{\diamond}(\bar{t},\bar{q})$ for $\bar{L} \in \left[\frac{\nu(\bar{q}-\underline{q})+\delta(\bar{q}-q_i)}{3\delta+\bar{q}-\underline{q}}, \frac{\bar{q}-q_i}{3}\right]$, which suggests that $\Pi_e^{\diamond}(\bar{t}^\star,\bar{q}) \ge \Pi_e\left(\bar{t}^\star,\underline{q},\bar{q}\right) = \Pi_e^{\circ}(\underline{q})$, i.e., the ignored constraint holds. Therefore, as $\Pi_e^{\diamond}(\bar{t},\bar{q})$ is decreasing in $\bar{t}$, we must have $\bar{t}^\star$ as the solution to (3.5).

Indeed, our claim follows from the fact that

$$\Pi_e\left(\bar{t},\underline{q},\bar{q}\right)-\Pi_e^{\diamond}(\bar{t},\bar{q}) = \frac{1}{8}\left[\left\{\left(1-\frac{\bar{q}-\underline{q}}{\delta}\right)^2+8\right\}\bar{L}+\frac{\left(\bar{q}-q_i+\nu/\delta(\bar{q}-\underline{q})\right)^2}{\bar{L}}\right.$$
$$\left.+2\left\{\left(1-\frac{\bar{q}-\underline{q}}{\delta}\right)\left(\bar{q}-q_i+\frac{\nu}{\delta}(\bar{q}-\underline{q})\right)-4(\bar{q}-q_i)\right\}\right]$$

is decreasing in $\bar{L}$ by examining its derivative

$$\frac{\partial}{\partial\bar{L}}\left\{\Pi_e\left(\bar{t},\underline{q},\bar{q}\right)-\Pi_e^{\diamond}(\bar{t},\bar{q})\right\} = \left(1-\frac{\bar{q}-\underline{q}}{\delta}\right)^2+8-\frac{\left(\bar{q}-q_i+\nu/\delta(\bar{q}-\underline{q})\right)^2}{\bar{L}^2}$$
$$\le 9\left[1-\frac{\left(\bar{q}-q_i+\nu/\delta(\bar{q}-\underline{q})\right)^2}{\left(\bar{q}-q_i\right)^2}\right]\le 0,$$

where we again use the fact that $\left|1-\frac{\bar{q}-\underline{q}}{\delta}\right| \le 1$.

- When $\bar{L} \ge \frac{\bar{q}-q_i}{3}$, we have

$$\Pi_e\left(\bar{t},\underline{q},\bar{q}\right) = \left[\left(2-\frac{\bar{q}-\underline{q}}{\delta}\right)\bar{L}+\frac{2}{3}\left(\bar{q}-q_i\right)+\frac{\nu}{\delta}(\bar{q}-\underline{q})\right]^2/(8\bar{L})$$

$$=\frac{1}{8}\left[\left(2-\frac{\bar{q}-\underline{q}}{\delta}\right)^2\bar{L}+\frac{\left(\frac{2}{3}\left(\bar{q}-q_i\right)+\frac{\nu}{\delta}(\bar{q}-\underline{q})\right)^2}{\bar{L}}+2\left(2-\frac{\bar{q}-\underline{q}}{\delta}\right)\left(\frac{2}{3}\left(\bar{q}-q_i\right)+\frac{\nu}{\delta}(\bar{q}-\underline{q})\right)\right],$$

which is decreasing in $\bar{L}\leq\frac{\frac{2}{3}\delta(\bar{q}-q_i)+\nu(\bar{q}-\underline{q})}{2\delta-(\bar{q}-\underline{q})}$ and increasing $\bar{L}\geq\frac{\frac{2}{3}\delta(\bar{q}-q_i)+\nu(\bar{q}-\underline{q})}{2\delta-(\bar{q}-\underline{q})}>\frac{\bar{q}-q_i}{3}$.
In particular, if $\nu+\delta\geq\frac{\bar{q}-q_i}{3}$, or equivalently $\bar{q}-q_i\leq 3(\nu+\delta)$, we have

$$\begin{aligned}\Pi_e\left(1,\underline{q},\bar{q}\right)-\Pi_e^{\circ}(\underline{q})=&\frac{1}{8(\nu+\delta)}\left[2(\nu+\delta)+\frac{2}{3}(\underline{q}-q_i)-\frac{1}{3}(\bar{q}-\underline{q})\right]^2-\left(\underline{q}-q_i-\nu\right)\\ \leq&\frac{1}{8(\nu+\delta)}\left[2(\nu+\delta)+\frac{2}{3}(\underline{q}-q_i)\right]^2-\left(\underline{q}-q_i-\nu\right)\\ =&\frac{1}{8(\nu+\delta)}\left\{\left[2(\nu+\delta)-\frac{1}{3}(\underline{q}-q_i)\right]^2-(\nu+\delta)(\nu+3\delta)\right\}\leq 0,\end{aligned}$$

for $6(\nu+\delta)-3\sqrt{(\nu+\delta)(\nu+3\delta)}\leq\underline{q}-q_i\leq 3(\nu+\delta)$. Therefore, there uniquely exists $\bar{t}^{\star}\in(0,1)$ such that $\Pi(\bar{t},\underline{q},\bar{q})\leq\Pi_e^{\circ}(\underline{q})$, if and only if $\bar{t}\geq\bar{t}^{\star}$.

Furthermore, it is straightforward to verify that

$$\begin{aligned}\Pi_e\left(\bar{t},\underline{q},\bar{q}\right)=&\left[\left(2-\frac{\bar{q}-\underline{q}}{\delta}\right)\bar{L}+\frac{2}{3}\left(\bar{q}-q_i\right)+\frac{\nu}{\delta}(\bar{q}-\underline{q})\right]^2/(8\bar{L})\\ <&\left[2\bar{L}+\frac{2}{3}\left(\bar{q}-q_i\right)\right]^2/(8\bar{L})=\left[\bar{L}+\frac{1}{3}\left(\bar{q}-q_i\right)\right]^2/(2\bar{L})=\Pi_e^{\diamond}(\bar{t},\bar{q}),\end{aligned}\tag{3.27}$$

which immediately suggests that $\Pi_e^{\diamond}(\bar{t}^{\star},\bar{q})\geq\Pi_e\left(\bar{t}^{\star},\underline{q},\bar{q}\right)=\Pi_e^{\circ}(\underline{q})$, i.e., the ignored constraint holds.

Therefore, as $\Pi_e^{\diamond}(\bar{t},\bar{q})$ is decreasing in $\bar{t}$, we must have $\bar{t}^{\star}$ as the solution to (3.5). □